

**PROBLEM 4.** [5 pts]

Consider a triatomic molecule,  $XY_2$ . Its bonds are based on sp hybridized orbitals.

- What is the geometrical shape of the molecule? [1 pts]
- Somewhere within the whole series of the rotational energy levels of the molecule, there are 3 consecutive rotational levels that have energies of 56, 72, and 90  $\text{cm}^{-1}$ . Determine the rotational constant  $\tilde{B}$  (in units of  $\text{cm}^{-1}$ ) and the J values of these levels. Note that when using  $\text{cm}^{-1}$  as energy unit  $hc=1$ . [2 pts]
- Now the atom X is replaced by one of its heavier isotopes. Can the rotational spectrum be used to find out the location of X within the molecule and how? [2 pts]

**PROBLEM 5.** [6 pts]

Give a concise, precise description of

- an intrinsic semiconductor at  $T=0$  and at  $T>0$ , [2 pts]
- the functioning of a donor doped semiconductor crystal, [2 pts]
- and, the occurrence of a depletion zone in a p-n junction. [2 pts]

**PROBLEM 6.** [4 pts]

Consider a simple 3D square lattice with the atomic lattice distance equal to a.

- Calculate the volumes of the Wigner Seitz cell and first Brillouin zone cell. [2 pts]
- Consider the planes described by the Miller indices (2,3,1). Determine the distance between these planes. [2 pts]

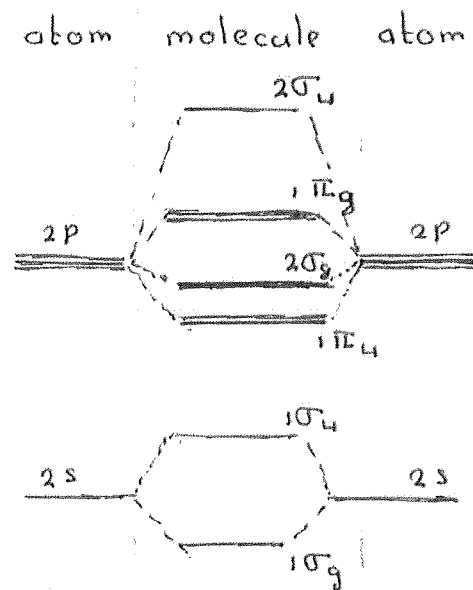
**PROBLEM 7.** [8 pts]

Consider a 2D free-electron metal with a simple rectangular lattice with the atomic lattice distances being a and 2a, respectively. The full crystal as a whole is square-shaped with sides of length L. L is equal to  $10^5$  a. To describe the electron gas traveling waves are used. Their wave function is given by:  $\psi = Ae^{ik_x x} e^{ik_y y}$  with  $k_i = \frac{2\pi}{L} n_i$  and  $i=x,y$ .

- Show that  $\psi$  meets the periodicity (or Born-von Karman) condition. [1 pts]
- Use to the Schrödinger equation to find the expression for the energy  $E_n$  of the free-electron gas with n defined as  $n = \sqrt{n_x^2 + n_y^2}$ . [1 pts]
- Each atom in the crystal donates one electron to the free-electron gas. Determine the Fermi energy in units of  $\frac{\hbar^2}{ma^2}$ . [2 pts]
- How does to Fermi energy compare to the lowest reciprocal lattice energy [2 pts]
- In reality the 2D crystal is not infinitesimal thin but has a certain thickness d. Estimate the maximal thickness d for which the crystal may still be considered to be a 2D free-electron metal. [2 pts]

**PROBLEM 1.** [7 pts]

Consider the  $B_2$  molecule. The electronic configuration of a single B atom is  $1s^2 2s^2 2p$ . The figure shows the relevant molecular orbital energy level diagram.



- Redraw the figure and indicate the electronic population of the molecular orbitals, use  $\uparrow$  and  $\downarrow$  for spin up and down, respectively. [2 pts]
- Which one of the following molecules  $B_2^-$ ,  $B_2$ , and  $B_2^+$  has the highest dissociation energy, and why? [1 pts]
- Give the electronic configuration of the  $B_2$  molecule, [1 pts]
- Determine the term symbol of the ground electronic configuration, [3 pts]

**PROBLEM 2.** [5 pts]

Consider a heteronuclear diatomic molecule AB. The bonding orbital of the molecule is given by  $\psi = 3\phi_A + 2\phi_B$ . The wavefunctions  $\phi_A$  and  $\phi_B$  are real.

- Normalize the wavefunction for the case that the overlap integral is 0.25. [2 pts]
- Determine the charge imbalance between A and B. [1 pts]
- For the case of the overlap integral being 0, determine the wavefunction of the antibonding orbital. [2 pts]

**PROBLEM 3.** [5 pts]

Consider now a solvated molecule in its ground vibrational state. The lowest molecular orbitals are sketched in the figure.

- Give a description of the (sequence of) processes leading to phosphorescence after photon absorption. Redraw the figure and include (schematically) all the processes from absorption to phosphorescence. [3 pts]
- The molecules are to be used as a laser medium, indicate which transition you would use as laser transition, and why. [2 pts]

